

## Current Drive in a Ponderomotive Potential with Sign Reversal

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Noninductive current drive can be accomplished through ponderomotive forces with high efficiency when the potential changes sign over the interaction region. The effect, which operates somewhat like a Maxwell demon, can be practiced upon both ions and electrons. The current-drive efficiencies, in principle, might be higher than those possible with conventional rf current-drive techniques. It remains, however, for us to identify how the effect might be implemented in a magnetic fusion device in a practical manner.

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Radio frequency (rf) waves can drive currents efficiently in plasma, through resonant wave-particle interactions, in several frequency regimes, including the lower hybrid [1], the electron-cyclotron [2], and other frequency regimes [3]. In the ion cyclotron frequency regime, low-frequency waves can resonantly drive either electrons [4] or ions [5] to produce current-drive effects. All of these effects enjoy considerable experimental verification, but the efficiency in any resonant rf current drive is limited in that it is a diffusive process, with low-energy particles propelled to higher energy to drive current. Kinetic energy to the particles must be provided by the wave, unless, in an inhomogeneous plasma, a population inversion exists along a diffusion path that connects to a low-density low-energy region [6]. Otherwise, these current-drive schemes tend to be less efficient than is current drive with a dc electric field; a dc electric field drives all electrons in the same direction, so that momentum is imparted also by braking those electrons traveling against the force field, thereby also extracting kinetic energy.

The question remains whether higher efficiencies can be achieved via nonresonant so-called “ponderomotive” forces, which have been studied in a number of contexts [7–14], including driving plasma current [8,9]. Litwin [10] suggests that there is cancellation that reduces the so-called alpha effect relied upon by others [8]. What we show, however, is that the current-drive effect might be achieved efficiently in an inhomogeneous magnetic field, where an important asymmetry of the ponderomotive potential may be exploited [7], rather than in the absence of a magnetic field or in the presence of only a uniform magnetic field, where this effect cannot be realized.

To show the current-drive effect in an inhomogeneous field, suppose a plasma is immersed in a magnetic field  $\mathbf{B}_0$  largely in the  $z$  direction, with some variation in  $z$ , so that  $\mathbf{B}_0 \approx B_0(z)\hat{\mathbf{z}}$ . The cyclotron frequency of a particle with charge  $e$  and mass  $m$  is then  $\Omega(z) = eB_0(z)/mc$ . Suppose an electric field of the form  $\mathbf{E}_{\text{rf}} = \hat{\mathbf{x}}E_x(z)\sin\omega t$ , with the consistent magnetic field given by  $\partial\mathbf{B}_{\text{rf}}/\partial t = -c\nabla \times \mathbf{E}_{\text{rf}}$ . This imposed field is assumed for simplicity in demonstrating the current-drive effect; a more precise

calculation, beyond the scope of this effort, should consider the propagation of realizable fields, obeying the plasma dispersion relation. In the magnetic field and electric field imposed here, the charged particle experiences an average ponderomotive potential, which may be written as

$$\Phi(z) = \frac{1}{4}mv_{\text{osc}}^2(z)\frac{\omega^2}{\omega^2 - \Omega^2(z)}, \quad (1)$$

where  $v_{\text{osc}}^2 \equiv (eE_x/m\omega)^2$ . A charged particle, in the ponderomotive potential (1), exhibits reversible motion if the beat frequency  $\omega - \Omega$  changes little in a period, i.e.,  $v_z\Omega'(z)/(\omega - \Omega)^2 \ll 1$ , where  $v_z$  is the particle velocity along the magnetic field [12]. Near the cyclotron resonance  $\Omega = \omega$ , this condition is clearly violated, resulting in chaotic motion of the particle. The characteristic width of the resonant interaction region is given by  $\delta z = \sqrt{L_B v_z / \omega}$ , where  $L_B$  is the characteristic scale of the dc magnetic field.

Note that the potential given by Eq. (1) is asymmetric in the  $z$  direction, as shown in Fig. 1. Ponderomotive potentials of this form have been proposed for rf confinement of plasma (for a review, see Ref. [7]) and stabilization of low-frequency modes in magnetically confined plasmas [13], as well as for isotope separation in plasmas composed of multiple ion species [14]. These applications make use of the enhancement of the potential near the resonance, rather than the sign reversal. What we show here is that it is precisely the sign reversal in the potential that can be exploited to produce a nonresonant current-drive effect with high efficiency.

Assume a  $z$  dependence of electric field  $E_x(z)$  and magnetic field  $B_0(z)$  such that, at  $z = 0$ ,  $E_x(z)$  has a maximum and  $\omega = \Omega$  (see Fig. 1). Define region I (say,  $z < z_1$ ), for which  $\omega < \Omega(z)$  and for which the particle motion is essentially adiabatic. Similarly, define adiabatic region III (say,  $z > z_2$ ), for which  $\omega > \Omega(z)$ . Define resonant region II,  $z_1 < z < z_2$ , for which the particle is essentially in resonance ( $z_2 - z_1 \sim \delta z$ ). Particles encountering the adiabatic regions may be reflected, whereas upon traversing the resonant region, they are subject to

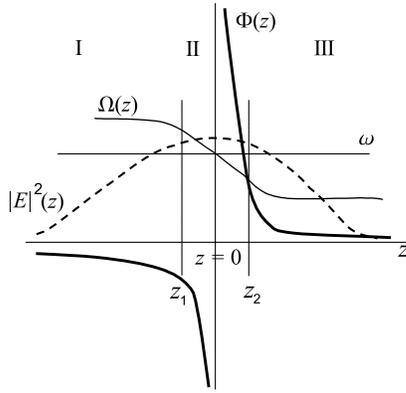


FIG. 1. Schematic of the field configuration. The electric field energy profile is shown by the dotted line and the ponderomotive potential  $\Phi(z)$  by heavy solid lines. The maximum of the electric field occurs at  $z = 0$ , which is also where the local gyrofrequency  $\Omega(z)$  equals  $\omega$ , the frequency of the applied electric field.

perpendicular heating. Particles impinging on the ponderomotive region traveling in the  $-\hat{z}$  direction can be reflected, but because of the sign reversal, particles encountering the barrier traveling in the  $+\hat{z}$  direction will be further accelerated precisely in the  $+\hat{z}$  direction. Thus, such a barrier, operating essentially as a Maxwell's demon, might be very efficient in generating current. The precise electric field profile should be important neither in the adiabatic region (so long as it is slowly varying) nor in the resonant region, which could even tolerate a minimum rather than a maximum on axis.

In addition to the ponderomotive force, a magnetic force  $\mathbf{F} = -\mu \nabla B_0$  accelerates particles along the dc magnetic field, where the quantity  $\mu = mu_{\perp}^2/2B_0$  is an approximate integral of particle motion [7], analogous to the adiabatic invariant of free gyromotion in a slowly varying magnetic field. (Here  $\mathbf{u}_{\perp} = \mathbf{v}_{\perp} - \mathbf{v}_{rf\perp}$  is the perpendicular velocity in the absence of the velocity oscillation,  $\mathbf{v}_{rf}$ , due to the rf field.) Note from Fig. 1 that both the magnetic ( $\mu \nabla B_0$ ) and ponderomotive ( $\nabla \Phi$ ) forces tend to drive particles in the  $+\hat{z}$  direction in the regions, where the ponderomotive potential is established (regions I and III). Inside region II, where the ponderomotive approximation does not hold, the force exerted on a particle also has the same sign, since it is primarily the magnetic force, with the magnetic moment  $\mu$  changing in time because of resonant interaction with the rf field. Because of these nonadiabatic effects, the integrated magnetic force,  $\mathbf{F} = -\mu \nabla B_0$ , does not quite vanish over a return of the particle to the same magnitude magnetic field, which produces a current-drive effect as well [15].

To calculate the current-drive effect produced by the rf barrier, imagine a region in space where the ponderomotive force is applied (see Fig. 2). Suppose that the magnetic field going in and out of this region is of the same magnitude and direction, although the field lines pinch

inside the region. In the pinch region, the cyclotron resonance is satisfied locally. Suppose also that collisions are negligible over the time a particle crosses the pinch region. Then, electrons (or ions) coming into this region are either reflected or transmitted, but in any event, the slowing down of the electrons occurs independently of the ponderomotive field. As such, what matters is only that, as a result of encountering the barrier, electrons go from velocity space location  $\mathbf{v}_1$  to location  $\mathbf{v}_2$ . If one knew the transformation function  $\mathbf{v}_2 = \mathbf{T}(\mathbf{v}_1)$ , one would be able to calculate the efficiency by simply averaging over all particles.

To calculate the current-drive efficiency, note that the region of current-drive excitation (Fig. 2) can be thought of as a current source. The source operates as a selective barrier. The number of particles in the velocity interval  $d^3v_1$  scattered off the barrier during time  $dt$  within the cross section  $dA$  is  $dN_B = f(\mathbf{v}_1)|v_{z1}|d^3v_1 dt dA$ , where  $f(\mathbf{v}_1)$  is the electron velocity distribution function.

Suppose the plasma is toroidal with period  $2\pi R$ . The instantaneous particle velocity  $\bar{v}_z$  in the  $\hat{z}$  direction changes as a result of collisions with background particles, so that it is a function of both initial coordinates and time, i.e.,  $\bar{v}_z = \bar{v}_z(\mathbf{v}, t)$ , where  $\bar{v}_z(\mathbf{v}, t=0) = v_z$ . Suppose the resonant particles are electrons. The current carried by  $dN_B$  electrons beginning with  $v_z = v_{z1}$  is then  $dI_z^{(1)} = edN_B \bar{v}_z(\mathbf{v}_1, t)/2\pi R$ . Then the current generated by  $dN_B$  electrons scattering from velocity space location (1) to velocity space location (2) is then given by the difference between the current generated over time by an electron beginning in location (2) and the current generated by an electron beginning in location (1). Under continuous scattering by a barrier from location (1) to location (2), the generated current can then be written as

$$J = \frac{e}{2\pi R} \int f(\mathbf{v}_1)[\chi(\mathbf{v}_2) - \chi(\mathbf{v}_1)]|v_{z1}|d^3v_1, \quad (2)$$

where  $\chi(\mathbf{v})$  is the current response function:

$$\chi(\mathbf{v}) = \int_0^{\infty} \bar{v}_z(\mathbf{v}, t) dt. \quad (3)$$

For example, for constant slowing down rate  $\nu$ , we get

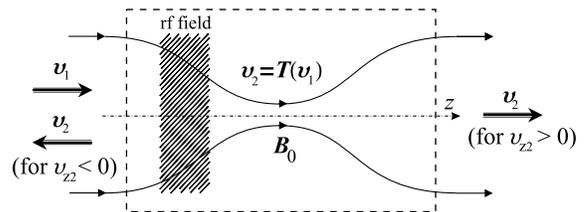


FIG. 2. Region of current drive excitation. The magnetic field going in and out of the excitation region is of the same magnitude and direction. The field lines return on themselves outside of the current drive region. A particle can be either reflected or transmitted by the rf barrier. The particle velocity  $\mathbf{v}_2$  after scattering is mapped by the nonlinear operator  $\mathbf{T}$  to the velocity  $\mathbf{v}_1$  before scattering.

$\chi(v_z, v_\perp) = v_z/v$ . For superthermal velocities, the velocity dependence of the collisions cannot be ignored; in the high-velocity limit  $\chi(v_z, v_\perp) \rightarrow (v_z/v_0)(v/v_{th})^3/(5 + Z_i)$ , where  $Z_i$  is the ion charge state,  $v_{th}$  is the electron thermal speed, and  $v_0$  is the collision frequency of thermal electrons [2].

The efficiency of a current-drive scheme is determined by how much current can be produced per unit power taken from an rf source. Perfect adiabatic reflection would generate current with no power dissipation, acting like a Maxwell demon, and thus violating thermodynamic laws. Irreversible heating inevitably accompanies the current drive, because particles transmitted through the barrier are stochastically heated as they traverse the resonance. This heating is additional to the antenna heating, which is neglected here. Then the power dissipated by the rf barrier per unit cross section is

$$P = \int f(\mathbf{v}_1) \Delta \mathcal{E}(\mathbf{v}_1) |v_{z1}| d^3 v_1, \quad (4)$$

where  $\Delta \mathcal{E}(\mathbf{v}_1) = m(v_2^2 - v_1^2)/2$  is the average irreversible energy gain of an individual particle as it scatters off the rf barrier. This calculation represents the full dissipation of the rf field, neglecting only the energy leakage if the mode were not standing, but had a finite group velocity. For a standing mode of the plasma, as we anticipate, the dissipation is simply equal to the energy gained from the particles passing through it, as calculated here.

The current-drive efficiency  $\eta = J/P$  can be determined precisely for a Maxwellian distribution of electrons in a straightforward if tedious calculation. However, one can give a very rough estimate of the efficiency by considering a model velocity distribution

$$f(\mathbf{v}) = n \frac{\delta(v_\perp - v_{\perp c})}{4\pi v_{\perp c}} [\delta(v_z - v_{z c}) + \delta(v_z + v_{z c})], \quad (5)$$

where the positive quantities  $v_{\perp c}$  and  $v_{z c}$  stand for the characteristic transverse and longitudinal electron velocities of the order of  $v_{th}$ . Substituting (5) into Eqs. (2) and (4) and using  $\chi(v_\perp, -v_z) = -\chi(v_\perp, v_z)$ , one gets

$$\eta = \frac{e}{2\pi R} \frac{\chi(\mathbf{v}_+) + \chi(\mathbf{v}_-)}{\Delta \mathcal{E}_+ + \Delta \mathcal{E}_-}, \quad (6)$$

where  $\mathbf{v}_\pm \equiv \mathbf{T}(v_{\perp c}, \pm v_{z c})$ , and  $\Delta \mathcal{E}_\pm$  stands for the energy change of a particle with initial velocity  $(v_{\perp c}, \pm v_{z c})$ . Since Eq. (6) does not depend on the actual mapping  $\mathbf{T}(\mathbf{v})$ , it is applicable for estimating the current-drive efficiency for a variety of mechanisms that heat and accelerate charged particles, as long as those processes can be described by a velocity space mapping.

How large is the efficiency of the current drive in a ponderomotive potential with sign reversal? The largest efficiency is achieved when the heating is minimized. To reflect adiabatically most of the particles coming from region III, suppose that the height  $\Phi_{max}$  of the ponderomotive potential is larger than the electron ther-

mal energy:  $\kappa \equiv \Phi_{max}/mv_{th}^2 > 1$ . For simplicity, we neglect the asymmetric magnetic mirroring effects; in the absence of the effects that we discuss here the magnetic effects, arising from  $\mu \nabla B_0$  forces, being symmetrical, must cancel. In principle, however, they could be included in the transmission function  $\mathbf{T}$ . The particles heated are then those that travel backwards (from region I) and pass through the resonance region. The transverse heating for these passing particles is approximately  $\Delta \mathcal{E}_\perp = (\pi/2)\sqrt{\Lambda} \Phi_{max}$ , where  $\sqrt{\Lambda} = \sqrt{L_B \omega / v_z}$  is the number of cyclotron orbits completed in traversing the resonant region. For  $\Lambda \gg 1$ , transmitted particles are mainly heated rather than accelerated, so substitute  $\chi(\mathbf{v}_+) = v_{z c}(v_{z c}^2 + v_{\perp c}^2 + 2\Delta \mathcal{E}_+/m)^{3/2} v_{th} v_0^{-1}$  and  $\Delta \mathcal{E}_+ = (\pi/2)\kappa\sqrt{\Lambda} m v_{th}^2$  into Eq. (6). For reflected particles, use  $\chi(\mathbf{v}_-) = v_{z c}(v_{z c}^2 + v_{\perp c}^2)^{3/2} v_{th} v_0^{-1}$  and  $\Delta \mathcal{E}_- = 0$ . The efficiency of the ponderomotive current drive is then

$$\frac{\eta_{PM}}{\eta_0} = \frac{2u}{\pi\kappa\sqrt{\Lambda}} [(v_{z c}^2 + v_{\perp c}^2 + 2\alpha\kappa\sqrt{\Lambda})^{3/2} + (v_{z c}^2 + v_{\perp c}^2)^{3/2}], \quad (7)$$

where  $\eta_0 = e/2\pi m R v_0 v_{th}$ , and  $v_{z c}$  and  $v_{\perp c}$  are measured in units of  $v_{th}$ .

To gain an appreciation for this efficiency, we compare the efficiency in this very approximate model of the ponderomotive effect to that achieved in the electron-cyclotron current-drive (ECCD) scheme [2]. In ECCD, electrons traveling in one direction are heated by the rf field in the perpendicular direction. Thus, take  $\chi(\mathbf{v}_+) = v_{z c}(v_{z c}^2 + v_{\perp c}^2 + 2\Delta \mathcal{E}/m)^{3/2} v_{th} v_0^{-1}$  and  $\Delta \mathcal{E}_+ \equiv \Delta \mathcal{E}$  for the electrons incident on the barrier with positive  $v_z$ ; and take  $\chi(\mathbf{v}_-) = -v_{z c}(v_{z c}^2 + v_{\perp c}^2)^{3/2} v_{th} v_0^{-1}$  and  $\Delta \mathcal{E}_- = 0$  for those incident with negative  $v_z$ . Using Eq. (6) in the limit  $\Delta \mathcal{E} \rightarrow 0$ , the ECCD efficiency is then given in dimensionless form by  $\eta_{EC}/\eta_0 = 3v_{z c}\sqrt{v_{z c}^2 + v_{\perp c}^2}$ . To estimate the ratio  $\eta_{PM}/\eta_{EC}$ , take  $\kappa = 2$  and  $\Lambda = 4$ ; for the normalized energy of 2D transverse motion, take  $v_{\perp c}^2 = 2$ ; and, since in the range of unity the ratio of the two efficiencies is insensitive to its precise value, take  $v_{z c} = 1$ . One then gets  $\eta_{PM}/\eta_{EC} \geq 2$ . What this crude calculation tells us is that the ponderomotive current-drive efficiency can be large, maybe even larger than the ECCD efficiency.

The effects calculated here apply equally well to ions as to electrons. Driving a minority species ion current in a plasma with two ion species can similarly lead to efficient current drive [5,16], and the ions driven by the ponderomotive effects suggested here will lead to similar current-drive efficiencies. To drive an ion current, consider two species of ions, one with charge state  $Z_\alpha$  times the other charge state. Suppose the ions drift relative to each other. In the frame of reference in which the ion current vanishes, the electrons will tend to follow the ions with the higher charge state, since they collide more often

with the higher charge ions, resulting in net current, density  $J = en_\alpha v_\alpha Z_\alpha (1 - Z_\alpha)$ . Since the current in a neutral plasma is frame invariant, the current appears in the laboratory frame as well. To induce a drift of momentum  $p_\alpha$ , by pushing ions from velocity space location (1) to velocity space location (2), requires power  $P_D$ , such that

$$\frac{p_\alpha}{P_D} = m_\alpha \frac{(v_z/\nu)_2 - (v_z/\nu)_1}{\Delta\mathcal{E}}, \quad (8)$$

where  $\nu = \nu_e + \nu_i$  is the sum of the collisional slowing down frequencies on electrons and majority ions, respectively. For minority species current drive, which has recently enjoyed experimental verification [17], minority ions are, say, cyclotron heated in the perpendicular direction, to velocity space location (2) from nearby location (1). Thus, we get  $p_\alpha/P_D = (3/2)m_\alpha(v_z/\nu) \times (v_i/\nu)$ , where the effect is maximized when  $v_e \approx v_i$ . For the case of ponderomotive barrier reflection, we get an induced drift  $p_\alpha/P_D = 2m_\alpha(v_z/\nu)/\Delta\mathcal{E}_\perp$ , where we again used a model two-delta function distribution (5), except that we assumed that the main current-drive effect came from reflection rather than heating. The main inefficiency occurs from the perpendicular heating, which is related to the maximum ponderomotive potential, as for electrons, by  $\Delta\mathcal{E}_\perp = (\pi/2)\sqrt{\Lambda}\Phi_{\max}$ . Comparing these two efficiencies, we can see that, if the ponderomotive potential is several times the minority ion temperature so as to reflect nearly all impinging minority ions, and if  $\Lambda$  is kept small (though still larger than unity), then the current-drive efficiency is of the order or can exceed the minority species current-drive efficiency.

What can be concluded is that the ponderomotive current-drive effect with sign reversal has an efficiency that might be on the order of the efficiencies of leading resonant rf current-drive mechanisms. But because this mechanism depends on very different physics, with very different parametric dependencies, there remains the enticing possibility that an optimized implementation of this effect could result in even higher rf current-drive efficiencies. These speculations come, however, with serious caveats. First, it remains for us to identify suitable plasma waves that can be excited in confinement devices of interest, producing a very localized, intense rf field, so that substantially all particles in one small flux tube can be reflected. Second, the dc magnetic field must change sharply enough that the parameter  $\Lambda$  is not too large; this parameter would be too large if the natural variation of the magnetic field in large fusion devices were relied upon (even for ions, and more so for electrons). These represent serious issues to be overcome before a useful current-drive scheme can be proposed for tokamaks. Among toroidal fusion devices, it may be more likely to realize this effect on a spherical torus, on a stellarator, or in the edge region of a tokamak, where there may be opportunities to sharply change the magnetic field strength along a magnetic field line. The effect might also find use on

linear devices or in regimes entirely apart from those normally considered for toroidal fusion devices.

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